

Closing Thurs: HW_6A,6B (7.4, 7.5)

Closing next Wed: HW_7A,7B (7.5, 7.7, 7.8)

Note: Friday is a university holiday (no class)

Exam 2 is next Thursday (Nov. 16th)

Covers 6.4, 6.5, 7.1-7.5, 7.7, 7.8

$$\begin{array}{r} x^2 + 3x + 2 \overline{) x^2 + 6} \\ \underline{-(x^2 + 3x + 2)} \\ -3x + 4 \end{array}$$

Entry Task: Evaluate

$$\int \frac{x^2 + 6}{x^2 + 3x + 2} dx$$

$$= \int 1 + \frac{-3x + 4}{x^2 + 3x + 2} dx$$

$$= \int 1 + \frac{7}{x+1} - \frac{10}{x+2}$$

$$= x + 7 \ln|x+1| - 10 \ln|x+2| + C$$

$$\frac{-3x + 4}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

$$-3x + 4 = A(x+2) + B(x+1)$$

$$x = -1 \Rightarrow 7 = A$$

$$x = -2 \Rightarrow 10 = B(-1) \quad B = -10$$

$$\text{CHECK: } -3x + 4 = (A+B)x + (2A+B)$$

$$A+B = -3 \quad \checkmark$$

$$2A+B = 4 \quad \checkmark$$

CHECK:

$$1 + \frac{7}{x+1} - \frac{10}{x+2}$$

Simplify
or plug in values

$$x=0 \quad 1 + 7 - \frac{10}{2} = 7$$

$$\frac{0^2 + 6}{0^2 + 3(0) + 2} = 7$$

$$\frac{0^2 + 6}{0^2 + 3(0) + 2} = 7$$

Example: Evaluate

$$\int \frac{x^2 - x + 6}{x^3 + 3x} dx$$

$$\frac{x^2 - x + 6}{x(x^2 + 3)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 3}$$

$$\Rightarrow x^2 - x + 6 = A(x^2 + 3) + (Bx + C)x$$
$$= Ax^2 + 3A + Bx^2 + Cx$$

$$x^2 - x + 6 = (A + B)x^2 + Cx + 3A$$

$$\Rightarrow A + B = 1 \Rightarrow B = 1 - A$$
$$C = -1$$

$$3A = 6 \Rightarrow A = 2$$

$$0 = 1 - A \Rightarrow B = -1$$

$$\int \frac{2}{x} + \frac{-x - 1}{x^2 + 3} dx$$

$$= \int \frac{2}{x} + \frac{-x}{x^2 + 3} + \frac{-1}{x^2 + 3} dx$$

$$= 2 \ln|x| - \int \frac{x}{x^2 + 3} dx + \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$$

$$\int \frac{x}{u} \frac{1}{2x} du$$

$$\frac{1}{2} \ln|u|$$

$$= 2 \ln|x| - \frac{1}{2} \ln|x^2 + 3| - \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$$

$$u = x^2 + 3$$

$$du = 2x dx$$

$$\frac{1}{2x} du = dx$$

Example:

$$\int \frac{x}{x^2 + 4x + 5} dx$$

IRREDUCIBLE!

$$x^2 + 4x + 5 = 0 \quad \text{HAS NO REAL SOLNS}$$

$$x = \frac{-4 \pm \sqrt{16 - 4(5)}}{2}$$

$$x^2 + 4x + 4 - 4 + 5$$

$$(x+2)^2 + 1$$

CHECK!

$$\int \frac{x}{(x+2)^2 + 1} dx$$

$$t = x + 2 \quad x = t - 2$$

$$dt = dx$$

$$\int \frac{t-2}{t^2+1} dt$$

$$\int \frac{t}{t^2+1} dt - \int \frac{2}{t^2+1} dt$$

$$u = t^2 + 1$$

$$du = 2t dt$$

$$\frac{1}{2} du = dt$$

$$\int \frac{t}{u} \frac{1}{2} du - 2 \tan^{-1}(t) + C$$

$$\frac{1}{2} \ln|u|$$

$$= \frac{1}{2} \ln|t^2+1| - 2 \tan^{-1}(t) + C$$

$$= \frac{1}{2} \ln|(x+2)^2+1| - 2 \tan^{-1}(x+2) + C$$

How to integrate

1. Look for simplifications/substitutions
3. Products/Logs/Inverse Trig → BY PARTS
Sin/Cos/Tan/Sec combos → TRIG
Quadratic (under a radical) → TRIG SUB
Rational Function → PART. FRAC.
2. If nothing seems to work, substitution.
($u = \text{inside}$, $u = \sqrt{\quad}$, $u = \text{trig}$, $u = e^x$)

Examples:

$$1. \int e^{\sqrt{x}} dx$$

$$t = \sqrt{x} \Rightarrow t^2 = x$$
$$2t dt = dx$$

$$\int e^t 2t dt$$

$$\int 2t e^t dt$$

$$= 2t e^t - \int 2e^t dt$$

$$u = 2t \quad dv = e^t dt$$
$$du = 2 dt \quad v = e^t$$

$$= 2t e^t - 2e^t + C$$

$$= \boxed{2\sqrt{x} e^{\sqrt{x}} - 2e^{\sqrt{x}} + C}$$

$$2. \int \frac{\cos(x)}{4 - \sin^2(x)} dx$$

$$t = \sin(x)$$

$$dt = \cos(x) dx$$

$$\int \frac{1}{4 - t^2} dt$$

$$\frac{1}{(2-t)(2+t)} = \frac{A}{2-t} + \frac{B}{2+t}$$

$$1 = A(2+t) + B(2-t)$$

$$t=2 \Rightarrow A = 1/4$$

$$t=-2 \Rightarrow B = 1/4$$

$$= \int \frac{1/4}{2-t} + \frac{1/4}{2+t} dt$$

$$= \left(-\frac{1}{4} \ln|2-t| + \frac{1}{4} \ln|2+t| \right) + C$$

DON'T
FORGET

$$= \left(-\frac{1}{4} \ln|2 - \sin(x)| + \frac{1}{4} \ln|2 + \sin(x)| \right) + C$$

$$3. \int \frac{3}{x - 2\sqrt{x}} dx$$

$$t = \sqrt{x}$$

$$t^2 = x$$

$$2t dt = dx$$

$$\int \frac{3}{t^2 - 2t} \cdot 2t dt$$

$$\int \frac{6t}{t(t-2)} dt$$

$$= 6 \ln|t-2| + C$$

$$= \boxed{6 \ln|\sqrt{x} - 2| + C}$$

$$1. \int e^x \cos(e^x) \sin^3(e^x) dx$$

$$\int \cos(t) \sin^3(t) dt$$

$$\int u^3 du$$

$$\frac{1}{4} u^4 + C$$

$$= \frac{1}{4} \sin^4(e^x) + C$$

$$t = e^x$$

$$dt = e^x dx$$

$$u = \sin(t)$$

$$du = \cos(t) dt$$